Math

Schools generally teach math in cookbook fashion. Often, the children are learning to use recipes without understanding why the recipe works, or even what the recipe is trying to accomplish. In this chapter, Susan shares some of the ways we have approached math in our home, starting with Jesse as a young child of five learning about the world of money.

Math and Money

Jesse, a budding consumer at garage sales and food co-ops, is beginning to really want to understand money values. What will this bunch of coins in his pocket buy at the hardware store? What is this business of pennies, nickels, dimes, quarters, and dollar bills? What's the sense of it? He's understanding clearly now that it's a good trick to only carry ONE dime rather than TEN pennies -- saves all that counting out and doesn't take up so much room in a pocket, a good invention. We've read a children's book on the history of money and he was quite intrigued. Could see that coins were a bit handier to tote about than goats or sheep for barter.

Then one day, as we were meandering through the Pittsburgh Carnegie Museum, we stumbled upon a small exhibit of old money. I boosted him up to see the odd bills with strange faces on them, the old lumpy coins, different sizes than ours. Talked a bit again about how money was an invention, something that's changed with time as people get new ideas, or as new governments take over. Then Jesse pointed out what seemed to be some broken or cut coins. We read the accompanying card and discovered we were looking at the original U.S. quarter! A silver dollar CUT into 4 bits! The pieces were shaped just like the "quarters" Jesse knew from our Sunday morning waffles. What a good laugh we had over this all. And Jesse on his own said that probably after a while they just decided to make quarters a separate round shape so that people would stop cutting up all the dollars. So Jesse now has a real visualization of what "four quarters equals one dollar" really means -- it's quite literally just like our four sections of waffle making one big round one. And they actually used to cut them up! What serendipity is possible with this homeschooling, the possibilities of coming upon these gems that help everything fall into place in new ways!

Now contrast this incident with this picture of a "wonderful" kindergarten that a friend enthusiastically described to me right around the same time as this museum visit, and you might get a clearer picture of how our learning at home is unique and so special. The kindergarten teacher, it seemed, had just completed a unit on "Money" with the class. I'm sure they did their share of xerox sheets on matching coin values, maybe

used plastic coins or maybe dollar bill stamp pads. The grand culmination of the "unit" was that the teacher actually I[took the class down to the supermarket, and they each got to buy a doughnut for a dime! I'm sure it was a "cute" field trip, with the children probably walking along the road in double file holding hands, looking a bit awed at being out of the building and in the REAL WORLD during school hours. But it sure doesn't excite me like that day in the museum with Jesse, and I doubt those kids learned half so much about consumerism or the mathematics of money, as Jesse did counting out his own nickels and quarters and dimes and dollars.

Approaching Math Through History

I'm wondering if any other homeschooling families have ever thought of the HISTORY of mathematics as being a source of possible interest to a non-math type of person (maybe you, or maybe your child), a way of beginning to look at mathematics in a new way with new eyes. It's an approach we have always used around here with Jesse and Jacob, and I have learned so much right along with the kids. Suddenly math, and even just "simple" arithmetic, becomes a long time-line of fascinating PEOPLE who tried, and tried hard, to make sense of their world with the best of their minds and the best tools of their times. It's no longer just stuff put in textbooks and achievement tests.

We began very simply with books that talked about probable first uses of numbers -- cave men making drawings on cave walls to show how many elk they had caught, shepherds tallying up sheep as they ran through a primitive gate. (Once when Jesse was 6 or so he helped us tally up our sheep during some maneuver, and we were surprised to realize that -- with sheep running about -- tallying IS indeed the only way to count them.) We looked into other earlier number systems -- the Egyptian, the Babylonian, the Mayan, and the Roman systems. Many of these were actually much easier for young Jesse to grasp -- he seemed to have a mental block for a good while about our particular numeral system. Although he could THINK wonderfully well with numbers in his head, he just couldn't remember a written out "5" from an "8," let alone "6" and "9." I often thought how painful arithmetic in school would have been for him, with its emphasis on numeral recognition and good "penmanship" in writing those numerals. It was reassuring to him to find that many other peoples of the world, at different times, had come up with different inventions for writing down numbers -- our present system wasn't God-given and set for all time, but was just the latest in a long line of tries.

It's even been intriguing to learn about the history of standard math operation symbols. My kids were delighted to hear stories about how the "x" sign was first used for multiplication, and all the various ways used for a century or two to show decimal

notation (and how the very idea of decimal notation at all was invented...). The equals sign ("=") has a history -- a certain mathematician felt it was the best symbol to use (others had been tried before him) because what could be more equal than two parallel lines. And there was quite a hullabaloo over how to write out fractions for a long time -- and we found that the Greeks didn't even want to consider fractions at all because they felt only the whole, natural numbers were perfect.

Now that Jesse is nine and Jacob is six, we've moved a bit farther in our math history. Jesse was just working on a section in his Miquon Math book (a workbook in a primary grade math series) introducing simple coordinate geometry -- and I can't understand why textbooks don't MENTION that these ideas have a STORY behind them, people who struggled to come up with these ideas. So we've just today read about Descartes and how he came up with his way of translating Euclid's geometry into algebra with his graphing system. Now we're not doing fabulously advanced work here, just graphing simple lines, but Jesse is grasping the idea that there is a lot more to it, that Descartes graphed circles and parabolas and ellipses, and that these methods paved the way for Newton to develop Calculus. Jesse isn't DOING calculus of course, but he's already hearing the word in non-scary contexts (we've just completed a good biography of Newton), as part of a STORY, and I know when he comes to study it in later years that he'll immediately remember the good stories of Leibnitz and Newton's "feud" over who really invented the idea in the first place. The topic will have a context for him, a time frame, a hook to place it in history. It won't just be textbook stuff with no past, not just a dry present of abstract problems to complete.

We've read a bit about Archimedes and Euclid and Pythagorus and other early Greek mathematicians, finding out how the Greeks' unique world view helped them to make the discoveries they did. Studying Math history is indeed one of the fine ways to view history (maybe better than the war-rulers-vanquished approach) -- it is not separate from the rest of the problems, or ideals, of the people of its time. We love reading about Pythagorus and his mystic group of students inventing numerology lore along with their concrete discoveries about triangle and square numbers and prime numbers. We're touched reading about Archimedes asking a conquering Roman soldier, about to run him through with a sword, to please wait just a minute so that he could finish the geometrical proof he was working out in the sand with a pointed stick. And of course the kids love the story of Archimedes running through the city after his bath shouting "Eureka! Eureka!" and love hearing about his fabulous cranks and cranes that could lift enemy ships right out the water and shake all the sailors out into the harbor. And when Jesse and Jacob come to more formally study geometry at some point, they'll think of Euclid gathering together all the theorems and proofs of three centuries of Greek geometrical thought and organizing them into his 13 books. And

they'll think of how Newton thought Euclid was just TOO obvious, and so he skirted over studying him thoroughly in favor of the more contemporary Descartes, only to be severely reprimanded by his mentor at Trinity College for this slighting. And of how when printing presses were first invented, Euclid's books were among the first to be published. Geometry won't just be some required course for some vague purpose like "getting into college," but a study tied to past stories (AND real experiences with compasses and rulers and blocks and hexagon tiles and all the geodesic models hanging from our ceiling).

And finding that early on Descartes and his mentor up in Holland tried to work out the solution to the problem of velocity of falling objects, not knowing that Galileo had just solved the problem down in Italy, let us see again what it might have been like to live in a world with slower communication systems than now, indeed with no real communication systems at all. Learning about math history doesn't keep just in the realm of math, but as John Holt often said of the serious historical study of ANYTHING, leads to wider understandings of all sorts.

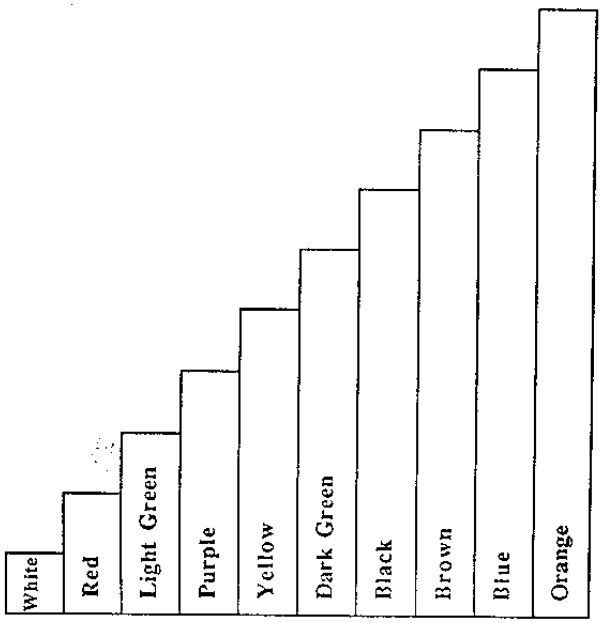
A fine author about the history of mathematics is Morris Kline. He's written profusely -- *Mathematics and the Physical World* and *Why Johnny Can't Add* are two I've read so far. I hear he has a new one out now specifically written to the "layman" about math history. He firmly believes that it has been one of the disasters of the education system to abstract math away from its history and roots in the real world. To teach math as if it were devoid of human interest or growth or change, and not linked to real physical problems out there in the world. Kline points out in his books also how intuitive many of the first mathematical discoveries were -- an intuitive hunch, a wondering, a dream image, that only later (often much later) was codified and formally proven and turned pristine and clean and pure and theoretical and abstract. He feels, too, that perhaps children would learn mathematics best in the order in which it was invented, that this would be a more natural unfolding at a more relaxed pace. And as for some of you folks who might be wondering, say, what good use negative numbers are, Kline would assure you that not only the "common folk" but even the greatest mathematicians of the day balked equally hard at the idea of negative numbers for centuries after they were first introduced from India. (Seems in India, negative numbers were first used very concretely as a way of talking about debts.) And the whole idea of irrational numbers so upset Pythagorus that he made all his students vow they would never tell anyone about these "unspeakables," these terrible blotches on his perfect number system -- and he surely wouldn't have liked the idea of negative numbers any better. Kline says if it took the great mathematical thinkers of the world so long to feel comfortable with these ideas, we should at least be a bit patient and understanding when a kid today feels a bit anxious around them or wonders what good they are.

Jacob (6 1/2) is the one who is surprising me lately with his math ruminating. Jacob is a daydreamy sort, who is often wandering about apparently doing "nothing." He's now letting me in on his world a bit more, and more often than not, he's been wondering about some math pattern he's been playing around with in his mind. Math thinking is indeed one of Jacob's favorite playthings. He'll stumble (often literally, Jacob can bump and tumble over anything or nothing) into the kitchen to announce happily that he knows how many hundreds there are in 2000! (I've checked the first grade "curriculum" for the district -- they aren't supposed to "get" to such big numbers until the next year...). He relates that he knew there were ten hundreds in one thousand, so there must be 20 hundreds in two thousand. Or I remember the time this last winter when we were driving home late at night from Pittsburgh and I was sure all the kids had peacefully fallen asleep in the back seat, suddenly Jacob's shy voice pipes up out of the dark with, "Do you know that ten hundreds is the same as twenty 50's???" Or he's always coming up with theories about square numbers, trying to find patterns in them (often erroneous, but still showing good thought). I often wonder what a school would have done with Jacob -- he still writes numerals backwards as often as forwards. (Though with chalkboard work he's getting better at it, and getting proud about his ability rather than balky about even trying.) The type of problems he's interested in wouldn't come up in the usual first grade textbook, and daydreaming is rarely tolerated in schools, let alone encouraged. And of course he never would have let his teacher know that he was daydreaming about NUMBERS of all things.

Sometimes as we play with geoboards or Cuisenaire rods or geodesic building sets or seeing what patterns we can make with our compasses, I think how Archimedes might have loved to use such toys, such playthings. These great mathematicians indeed WERE playful it seems -- and perhaps that's one of the most important ideas I hope to pass on to my kids in our math history learnings...

Starting Out With Cuisenaire Rods

Even before our children could recognize written numbers, they were discovering number concepts by, among other things in their world, using their Cuisenaire rods -- wooden blocks in 10 graduating sizes from a one centimeter cube to a 10 centimeter rod (1 x 1 x 10). You can build a "stairway" of rods from smallest to longest (see picture). Each length is always the same color -- red rods are always 2 cm.. long, blues are always 9 cm., etc. A small (74 rod) set of the rods runs about $7.foot There are no markings or numerals on the rods, which makes them more flexible in actual use.



I own a large set of Cuisenaire rods left over from a summer program I'd worked with years ago. I originally wanted to order them for the program because I'd read about them so much in John Holt's *How Children Fail* and *How Children Learn*. I couldn't figure out *what* in the world the rods looked like, and wanted to buy a set to find out. The kids I worked with that summer enjoyed making pretty patterns with the rods, but I never had a chance to see children really *use* the rods for mathematical purposes. One "open" first grade classroom I worked in while in college had Cuisenaire rods, but used them only for "color identification" exercises. The children were not allowed free access to them, and "real" math was done basically with standard workbooks. I wondered if my own children would extend the rods' possibilities, really use them for something more than just a pretty set of miniature blocks. (They *are* beautiful, I still find them a visual and tactile feast...)

Jesse certainly spent a very long time building and playing freely with rods as a toddler and young child. All the time, though, he was learning about these lengths and their relationships. As a 2 1/2 year old, he'd figured out that all reds, say, were always the same size. He worked with great concentration paving rod "roads" of different widths, fitting in rods exactly. The rods became barns, trains, families. He discovered stairway patterns, and for a good while made lots of variations on these:

- What would happen if a yellow rod was added to each step?
- Could stairways go up and down?
- Could stairways be built on top of stairways?

We talked a lot about which rods were longer, which shorter, which ones when put together equaled others. Our language gradually came closer to that of mathematics. We talked of "Black minus purple equals light green," or "3 light greens equals 1 blue."

Although it's hard now to remember just when we began relating the rods to number ideas, I know we just very naturally began talking in "Cuisenaire" terms throughout the day. Lengths of objects became "Oh, that's about as long as a *purple*," or "I think my fingers are about a *black* long." This usually followed by testing out with actual rods. Once, when Jesse was asked to pick up 10 things from the kitchen floor, and he had found 5, we laughed about how he'd needed to find an *orange* worth of things, and had already found a *yellow*, another *yellow* to go. A dozen was referred to as an *orange* plus a *red*, or 2 *dark greens*, or 4 *light greens*, or 3 *purples*. Lots of talk of halves and quarters, while measuring whole wheat flour for baking, led us to look into these relationships in the rods -- could Jesse find half an *orange*, half a *purple*, half a *blue*? This was quite intriguing to Jesse, as he found that some rods didn't have even halves, and we began discussing ideas of odd and even numbers. Jesse made stairways of all odd rods and all even rods. We've examined thirds, quarters, fifths, sixths. We've even used rods on a simple balance scale, having fun with balancing different combinations of rods.

When Jesse was too young to comfortably use written numerals to keep score in games, we'd sometimes use rods as a graphic way to show the game's progress. I remember well the very first time we hit on this idea. Like many of our very best ideas, this one was NOT planned in advance! Instead, it was a response to a problem we were faced with. We'd made ourselves a set of American Indian stick "dice" and were trying to learn how to play some games of chance with them. A book we were using suggested a scoring system -- 4 points for one combination of sticks, 6 for another, 8 for another, 10 for another. We began to play, and I quickly saw that using numerals to keep score meant next to nothing to Jesse. The game seemed a bit pointless, but Jesse's Indian enthusiasm made us keep at it anyway.

Suddenly an idea flew in -- why not use our Cuisenaire rods for score keeping? We poured out all of our rods and set to. I told Jesse that we could use any combination of rods that would equal the same number of *whites* (1 cm.) as our scores. So a score of 4 might be made with either *whites*, 2 *reds*, 1 *light green*, plus 1 *white* or 1 *purple*. Jesse caught on immediately and with great delight. We each kept score by lining up our rods along our own edges of the table; we agreed that going the width and length of the table equaled winning, and then we kept on going until our rod lines met. Anytime we saw we could exchange two or more rods for one longer length, we did it, if we wanted. I kept my own second game score entirely in *yellow* rods. There was an energy in the game now, active thinking and figuring, and delight in the varied

patterns we made. How much better than juggling numeral symbols that as yet had no meaning for Jesse.

Somehow I think it was important that we didn't let the rods become just props for other play. Although Jesse and Jacob both fantasized richly i[with the rods, we didn't mix Cuisenaire rods in with Lincoln logs, say, or use them as people for Tonka trucks, or stir them in with sand for pretend witch's brew. They were always a bit special, used only in certain places, carefully picked up, kept in certain containers. We found it helpful to either use them on rectangular trays or on an old secondhand coffee table with a raised edge (it's now officially called our "Cuisenaire table"). This kept the rods in one place, not falling scattered about, and also gave the boys a firm edge to line rods up against. We literally never put them away anymore, but always have them out ready for a moments use.

Once Jesse figured out how to show 100 with rods (10 *oranges*). Then he wanted to try to show one thousand. This took lots of work, as our set only has about 17 *oranges*. He worked very systematically and diligently -- the "200" square was made of all *yellow* rods (2 yellows equals 1 *orange*, the "300" square was made from 10 *blacks*, and 10 *light greens*, etc. These larger numbers have taken on quite a reality for Jesse. It reminds me of an experience I had as a 6 year old. I was lying in bed one night, counting as high as I could. I reached 100. Unsure what came next, I began saying "200, 300, 400, 500..." I stopped quickly, almost heady with the thought of how high I'd gone, but sensing that perhaps something might be wrong. But I had no way to test out my counting ideas, no model of how these bigger numbers really "worked." Jesse has rods to give him a very concrete way of imagining them.

Jesse seemed to really need, as well as enjoy, the concreteness of the rods. Written numerals meant little to him before almost age 6, even though we came upon numerals frequently on clocks, calendars, store advertisements, in books and games. The wiggles on paper just didn't carry meanings for him like the rods that he could tangibly feel and compare. With rods, addition combinations, say, are known by long experience and feel. They are all interrelated, observable, and real, rather than "math facts" to be memorized and drilled. I was amazed that the very first time Jesse saw written down addition problems this fall, he had absolutely no difficulty. I'd made a simple addition "concentration" game -- "1+5" could be matched to "3+3," or "2+4," or "5+1," or "6+0," etc. Jesse thought it was great fun, suggested other combinations I hadn't thought of. "Why couldn't you put '1+1+1' for 3?" etc. He lined up the cards in order, like he's done with rods, made a new game of adding up the *columns* of numbers, got out his rods to help him check his answers. The symbols are now easily understood and given meaning -- he is ready to transfer his very concrete ideas of number to numerical abstractions.

I can see that the rods have given Jesse, and now also Jacob, a very flexible approach to mathematics. Number ideas are playthings, delights, intriguing puzzles that fit together in ever varying new patterns. Mathematics is no dull workbook activity separate from his real living.

Doing Math Work

I was recently asked by some homeschooling mothers if Jesse, now 8 1/2, actually did math work readily, ALL ON HIS OWN, or did I have to force him to do it. I was a bit taken aback, and felt at a loss to answer. The question seemed too black/white, either/or, this/that. I had images of either complete abandonment (magically, some probably secretly hoping, creating a child who just LOVES math workbooks...) or a frantic, frazzled mother nailing her child to the dining room chair yelling "Do these ten multiplication problems OR ELSE!!!" To say Jesse goes to his math work ALL on his own completely by his own decision, seems an over-simplification of things. I've played a major role in helping Jesse see what math work might be appropriate, interesting and useful, and I keep a guiding, responsive hand in things. On the other hand, I don't need to yell or threaten or force, or even cajole him into his mathematics work. I'm trying here to sort out how this has come about.

Jesse, I think, knows several things about my views of mathematics. First, he knows it's an area I like and enjoy. I sometimes read books about math for my own pleasure (oh, not all the time, mind you, but a bit here and there), and I share good information I find with him. He also knows I sometimes leaf through some of the MANY books we have on math for kids and end up doing little geometric experiments, number line patterns, or puzzles myself -- just because I get inspired and intrigued by my reading. I've made paper models of pyramids, icosohedrons, cubes, and tetrahedrons. They know if I see a hexagon patterned sidewalk that I'm sure to remark enthusiastically about it. They also know that I snatch up geometric shaped paper-weights at any garage sale, and try to mentally figure out how to build models of "geodesic" shapes we come across. (We all especially got excited by the large cement geodesic banister heads outside the Capitol building in Harrisburg last time we were there lobbying for the Home Education Bill.) They know I've made our own geo-boards (square pieces of plywood with an evenly spaced array of nails dotting the surface -- you stretch rubber bands on to make all types of geometric patterns, show area, perimeter, you name it). I've spent time making our own Tangrams set out of wood (it's an old Chinese geometric puzzle, great fun for all ages). And my kids know that somehow I think Cuisenaire rods are important enough to warrant having their very own special table. It's always a sure thing that I'll buy games like "Quinto," or "Racko," or "Create a Cube," or one more set of dominoes at a yard sale, too. They don't think it's odd or

suspicious for me to give them Tri-man protractors and compasses as little Christmas gifts, and they are always thrilled to get new tape measures or folding rulers.

And STILL, with all this good mathematical "stuff" in the atmosphere, we also do use official math work books. Not the school's third grade text that we borrowed this year -- it was so deadly and boring and repetitive we could barely stand to flip through it. It seemed to shout out that kids have no minds, and only need rote, rote, rote, in exactly the same format day after day after day. We've chosen the *Miquon Mathlab Materials*.foot(Available from Key Curriculum Project, P.O Box 2304-C, Berkeley, CA 94702, phone 800-338- 7638.) (The books are a good buy -- for less than $30.00 you get 3 teacher's guide books and 6 workbooks, roughly geared towards first through third grades, but really more advanced.) These books aren't "fancy" -- they don't try to rely on colorful, jazzy illustrations (usually totally un-related to the math work) like the school text we abandoned. The math is sound and CREATIVE, respecting the fact that children are curious, active learners and like to see pattern in the world about them. Lots of work helping kids see the rhythms of numbers and measurement. Also lots of places where kids are asked to make up their own problems ("How many different names can you think up for the number 27?" etc.). The books also use Cuisenaire rods regularly in modeling new processes, so they were a natural for us.

Now -- how does Jesse use these books? At times we've used them just as another resource, one choice among many. Jesse would dip into them as he pleased, not worrying about doing the work in "order." This was fine for a while, but left him unable to understand some ideas that had been built up over time. Sometimes I'd suggest specific pages to him, either because I thought he'd find them especially intriguing or because they covered something I felt he ought to be learning something about.

At first we didn't notice or use a nice feature about the books. There's a chart on the back cover listing all the pages in the book by categories -- multiplication, addition, inequalities, fractions, mapping, etc., with little boxes for each page number. I realized that Jesse might work with more concentration in the books if he could mark in on the chart which pages he'd completed. It would be a way for him to keep track of his own progress, give him a graphic picture of where he'd been and was going, and what to expect ahead. Using the charts has been a real turn-around. Jesse loves seeing the boxes get filled in, even began inventing little games about armies advancing and conquering all the new territories, as his yellow crayon filled in the boxes for pages completed. We also began setting goals -- I asked Jesse when he hoped to have one book completed, and then to figure out how many pages he'd need to complete daily (not counting weekends) to reach his goal. For the third book Jesse set March 1st as his completion goal, found he'd need to do two pages a day for the next month and a

half to finish up -- and then proceeded to choose ON HIS OWN to work even on weekends so that he'd surprise us all by finishing up early! He sometimes decides to do more than two pages if he gets particularly excited by something, but doesn't use that as a reason to not do math work the next day, although he knows that would be OK by me.

When Jesse completed his book two weeks early he immediately wanted to dive into the next, and is now zooming along in the fourth book (correction -- as I'm retyping this he's finished the fourth book, early, and has begun the fifth...). He is still free to choose which sections of the books he works in, although he is now very diligent about being thorough and doing everything eventually. He explains that these out of sequence blocks on his chart are surprise raids into enemy territory -- we've been reading lots of books about the Civil War lately, so you'll have to excuse the battle imagery! I do think that his "playing" with the chart shows that he's found a way, on his own, to transform what might have been dull work into quite exciting play -- and his play makes the work his own.

Carol Wilson tells me her son, Luke, also enjoys this goal setting, chart-filling with the *Miquon* books. They seem to like, perhaps, having everything out in the open, a clear agenda they can understand, and have a share in shaping and pacing. I know, too, it helps Jesse to know that a number of our homeschooling friends also use these books. He likes hearing how other kids are doing with them, makes it all into a sort of social experience.

I am usually closely involved and in touch with what Jesse is doing with these books. I'm nearby, ready to offer help as needed, ready to share in discoveries and connections he's making. Ready to help him over or around a snarl or frustration that may come up. And I usually enjoy the subject, and make my own new connections, too. I had my "come-uppance" on this recently. I was heading out to milk the goats with two year old Molly, while Jesse and Jacob chose to stay inside. Jesse had decided he'd work in his math book while I was gone. As I left I thought to myself, "Ah! He's finally getting more independent in his math work -- terrific!" When I returned I tried to glance over what he'd done, feeling, as teachers in schools must, that since I hadn't been there to see his process of work and thinking, I must now check up afterwards to see if he'd done it all "right." I quickly saw a few gross errors, and tried to question Jesse about them, and "help" him. His reaction was to try to physically cover the page with his arms, and then even grab the book away from me. He became furious with me, and finally burst out, in choked tears, that if I wasn't going to be there while he was doing something, then I had no right to say anything about it afterwards. SO -- I now try to always be physically nearby, present in all ways, as he's doing math, discussing the work with him as he chooses, and Jesse's much happier for it. There's more of a sense of camaraderie about the work -- we are more like colleagues, out of

the roles of task-master/ child-academic slave, and onto both being students, ready to learn from the situation. He can sense more concretely my respect for his growing and his emerging ideas. I've responded to HIM.

But what do Jacob, just 5 1/2, and Molly 2 1/2, do while I'm helping Jesse with math? All depends. Mostly Molly is sitting on my lap (and, yes, sometimes kicking at Jesse's math book with a chubby, well-aimed foot). On some days, perhaps the best days, I get out manipulative math materials for Jacob and Molly -- cubical blocks, sum-stick, geoboards, puzzles, counting games, dice, etc., and they use these on the floor nearby while Jesse works at his desk. We actually do have old school desks for each of the boys -- yard sale finds again. I used to be perhaps "philosophically opposed" to having official desks for the kids, but have found that having a work surface to call your own, that's the right height for you, where you can store all sorts of books and treasures is good for the kids and they love their desk areas. Molly is clamoring for one too.

One day Jacob and Molly both happened to be building bigger and bigger sized squares with various tiles and blocks -- while Jesse was working on several pages dealing with square numbers! I was quick to point out the tie-in. Feels good when we can all be working in the same basic area at the same time, each in our own ways.

I think, too, that Jacob is less interruptive to Jesse's work if he's also had some good one-on-one time with me, if it doesn't seem like I'm just always pushing him aside or telling him to be quiet so I can do REAL work with Jesse. Jacob appreciates it when I won't let Jesse bother US when we are doing something special together. And Jesse needing me nearby doesn't mean he needs me ogling over his shoulder every second. I don't need to give him every bit of my attention. It's often enough that I keep in touch, discussing his work with him in between responding to Molly or Jacob, or in between sweeping the project room floor. Also Jacob and even Molly are now used to Jesse spending some time each day doing math work. They expect these shifts in work and play just as much as Jesse, and so don't balk (much) at them. Jacob is gradually working into a math time for himself -- sometimes using simple workbooks, or his own calculator, or Cuisenaire rods. And it's not odd now for Jacob to go about the house wondering aloud if two odd numbers added together will give you an odd or an even number for an answer, or coming up with a "tricky problem" for Jesse to solve.

Sometimes, of course, Molly is tired, hungry, or just plain out of sorts (maybe even rolling on the floor kicking and screaming after Jesse has yelled at her for knocking his carefully arranged Cuisenaire rods off his desk and all over the floor...). Then the most helpful thing I can do for everyone is to physically take Molly from the scene and calm her down somewhere else. Let her help me wash dishes in the sink (ah, water, the great soother...) get her a snack, get her a nap up in the bedroom where it's quiet. Jesse and Jacob both understand, then, that I can't be available to them for a bit.

We're all gradually learning the balancing act of living with a passionate little person who wants to be in on EVERYTHING. Not always easy, but very exciting to see how she's growing and learning too. The other day during Jesse's math time she spent at least half an hour stretching rubber bands very carefully on a geoboard, then watching to see if I could duplicate what she had done on my larger board. She made purposeful patterns -- little squares next to big ones (Mommy squares with their baby squares, she said...), triangles stretching magically into rectangles. She also loves our Cuisenaire rods, lines them up in rows and patterns, and never even TRIES to eat them. Molly even sometimes wants to do "her" workbooks, meaning she wants to scribble all over the pages of an old free workbook we picked up somewhere. So even her math time is coming along too.

One other thing about Jesse and math -- he has an important math JOB in our family. He's the official treasurer for our newsletter, and is in charge of filling out deposit slips for our newsletter checking account. Lots of real adding and multiplying here and a real need for accuracy. (He now readily accepts the idea of checking over his calculations, since we once received a notice from the bank saying that we actually had deposited $8.00 more than Jesse had recorded on the slip!) Jesse takes on this work seriously and very happily, feeling very adult (for me it was pure drudgery and I couldn't stand doing it, "red tape" never being my forte). This work has also brought about many good discussions of how checking accounts really work, how banks function, why forms are made as they are, etc. I'd urge any homeschooling family to consider giving a child a real family job that uses math calculating as part of it. I know that Barbara McMillan mentioned once that when her family went on long trips, Jaime's job was figuring out total mileage and how much was spent on gas. Kids could help pay bills or balance check-books. The possibilities are endless, and what may seem like boring routine paperwork for us, may be an exciting in-road into the adult world for our children. So, I don't think our math work at home is coerced or narrow, but neither is it totally "free form" in the sense of having no guidance from me. I've helped set a "math appreciation" atmosphere just as many parents actively create a rich musical atmosphere in their homes. I've worked to help the kids feel that it's just an expected tradition that we do some math work each day, just in the same way that, say, we eat three regular meals. And, like our meals that we all share together and have a good time at, so too we're sharing together in these math times. Rather than it being a fight for me to force the kids to "do their math work or else," we're ALL taking part, ALL learning, and ALL in it together. Makes a difference.

Using a Math Textbook

I have taught myself simple statistics and three computer languages by working through beginning textbooks. In each case, I had a real reason for learning what

the textbook had to offer. I had a problem that I wanted to solve, and the textbook enabled me to learn what I needed to know in order to solve the problem.

I once saw a television interview (the "Nova" show on public television) with one of the most famous physicists of our time, Richard Feynman. He described how he had learned calculus as a young boy. He had gone to the library and picked out *Calculus for the Practical Man*. When he took it to the checkout counter, the librarian challenged him. "Why are you taking out this book?" and Feynman lied, thinking the librarian wouldn't believe that he was taking it out for himself, "I'm taking it out for my father!" Later, on the recommendation of his high school physics teacher Feynman worked through an advanced calculus textbook on his own which gave him a slightly different knowledge of calculus than that of his fellow physicists. Later, he would often be able to solve equations that stumped everyone else, not because he knew a better way, but because he knew a different way.

The math textbooks written by John Saxon have been a favorite with many homeschoolers. Jesse is just starting to use *Math 76* (for sixth or seventh graders) which Saxon wrote with Stephen Hake.foot In issue number 20 of *Pennsylvania Homeschoolers*, Nathan Williamson, a ten year old homeschooled boy wrote:

> **There are many math books. They all have math in them. But the one I like is Saxon Math. I like the way Saxon Math programs the lessons and the problems so I can find where I'm at. I was never good in math. I was nine and I was in third grade math. But now I'm ten and I'm doing sixth grade math. And I feel good about me!**

Textbooks -- many of us use them, but sometimes we feel "guilty" about using or over-using them. Guilty that we are turning our homes into miniature schools of just the sort we've always complained about. Sometimes we may forget that we do have freedom -- we can USE these books HOW we want to. They can be our resources, helps, references, idea starters -- they don't have to be the end all and be all that they so often are in schools.

We use a math textbook with Jesse, always have, but I'm so glad we are free to use it as WE see fit, and not feel we need to slavishly follow it problem for problem without thought. I always feel it's most useful to us when we use it as a springboard to other real life math problems -- problems WE want to solve and work on.

A few days ago Jesse noticed that the next section in the text was on reading different types of graphs, something that has always been easy for him. He said right off, "Oh, I

know they always have reading graphs on achievement tests," meaning that he therefore wanted to work on them a bit so he'd be all set.

But the graphs were so insipid, so downright stupid. Graphs of how imaginary students in a fifth grade class got to school (bicycle, walk, or bus), graphs of the favorite colors of 5th and 6th grade students, graphs of heights of Leo, Pam, Rob, Kim, and Jay (pretend members of a pretend 5th grade class). A few were a touch more intriguing -- a graph of the growth of a baby mouse, or graphs of breathing rate and pulse rate after exercise.

But they just were not something we could exactly sink our teeth into. And I realized that we didn't HAVE to feel bound at all to doing work with these contrived graphs -- Jesse could make his own on a topic of interest to him. He could graph something he wanted to find out about, and possibly see how graphs actually are useful in sorting out information.

That same day we somehow stumbled into rummaging through our Almanac, something both boys really enjoy. I can't even remember now quite how it got started, but soon we were looking for answers to questions we had about the U.S. Census, and soon Jesse was grabbing the book and pouring over all the charts and tables about population growth in the U.S. Much more complex chart reading here than his math text offered -- and here we knew the statistics were REAL (we have our doubts about the accuracy of the math book problems -- sometimes we've felt burned when we've found that their "statistics" were totally made up and phony).

I told Jesse that his challenge was to make a graph using any of the information he found in the Almanac to make his own bar graph. He chose to show the changes of population in the original thirteen states from 1790 (the time of the first census) to 1900.

In an hour he was done -- and proud! And I think he learned MUCH more than he would have had he studiously completed the entire chapter in the text about graphing. He had to decipher charts and tables of statistics, and sort out what information was relevant to his chosen topic. Then he had to decide on a format to use -- where to list the states, where to list the populations, how to differentiate between the 1790 and the 1900 statistics, how to set a reasonable scale (not so large that he'd need three sheets of paper to show his results, not so small that the graph would take up only 4 tiny blocks on his graph paper...), and more.

And the most valuable lesson came as he was actually filling in the blocks with yellow and blue -- he began realizing with excitement how a graph really SHOWS information in a new way. As Jacob said while watching it all, "Just reading it in a

table in the Almanac doesn't let you SEE it all at once." The graph gave him a picture (isn't that what "graph" means anyway?) of his results, and he marveled over his new findings for a good while. "Hey, look, you can just SEE that New York really took off, but Virginia sure lost its place as number one," and "Now you can see why I needed to go all the way up to 7 million," and "Just look how some states hardly grew at ALL."

The idea has really taken off, and Jesse has now made several more graphs, all with the same enthusiasm. He's graphed the population growth of Pennsylvania from 1790 to the present, showing the population recorded by every census. He then made a graph of Georgia's and Florida's growth, careful to use the same scale as his Pennsylvania graph so he could really compare results. He's pored over these growth curves with a historian's eye, wondering why certain times saw large boosts in population, why other times growth slowed down.

He also made a very useful graph of new and renewal subscriptions to *PA Homeschoolers*, using data collected over the last two years. He already handles the checking account deposits, so this was a natural for him.

Now in all honesty, I probably wouldn't have thought of the idea of suggesting Jesse make his own graphs if there hadn't been that graphing unit in Jesse's book. That was the good use of the book. It got an idea going. So glad we let the math text book be a leaping off place today.

Math in the Real World

We're finding out more and more that all branches of math that we've studied don't just stay textbook problems for us. We're beginning to see the math that surrounds us all the while, and Jesse's growing ability and understanding of arithmetic is helping him do more with the real world.

We recently took a family trip all through the Southeast of our country, and "math" took place every day. Jesse and Jacob were often our navigators, and often estimated how long a certain drive should take -- which meant adding up mileage and figuring out the maps' scale, then figuring how fast we were driving, then allowing in for stops for meals or rest stops. Or Jesse figured out the average price of meals -- he'd heard endless discussions about the relative prices of steak houses and salad bars and fast food joints!

We saw the Gateway Arch Monument in St. Louis and talked and read about catenary arches and the types of curves made when you suspend a chain from two points, and saw how the whole arch was made up of gently decreasing equilateral triangles. We

also felt first hand the full 18 inches of allowable "sway" to the arch as we looked over the city of St. Louis on that VERY windy day! We tried to mentally calculate if the Arch was as tall as the US Steel Building in Pittsburgh.

On one of our last days on the road, Jesse figured out the average distance we'd travelled each day -- and could also see how meaningless such an average could be. "One hundred miles a day" was the proper answer to the problem, but that told nothing about the six or seven hours pushing for as many miles as possible one day, versus a quick half hour out to a beach another. (I used the chance to mention how so MANY averages mean just as little -- the *TELLS* test "average score" for the whole state of Pennsylvania tells us nothing at all about any individual child's actual score.)

Math in Real Books

Speaking of averages, Jesse recently read with great delight a book called *What Do You Mean by Average?*, about a girl who was trying to win a school election campaign with the slogan that she was the "average" girl in every way. The book brings in all the different ways to calculate averages, and how they all tell us different sorts of things. The book was a far cry from a math textbook -- it's amazing how many good math books there are to READ. And amazing how if an idea is tied to a STORY, then the idea sinks in and is remembered. We also had enormous math fun recently with a Newberry Award book, *The Phantom Tollbooth*, by Norman Juster. I'd owned the book for years but we'd never opened it until I realized the same author had written the delightful math fantasy picture book, *The Dot and the Line* (another math must). The book's basic plot is a sort of *Alice in Wonderland* journey to free two princesses ("Rhyme" and "Reason"), and at one point the young hero must travel through Digitopolis, the Kingdom of Numbers. Here we met a Dodecahedron character, who appropriately enough had ten FACES on his geometric head (we all made our own paper models of dodecahedrons with a different face drawing on each "face" of the solid -- don't think they'll ever get mixed up on what that math term means!) The Dodecahedron spouted out such problems as:

> Why, did you know that if a beaver two feet long with a tail a foot and a half long can build a dam twelve feet high and six feet wide in two days, all you would need to build Boulder Dam is a beaver sixty-eight feet long with a fifty-one-foot tail?[1]

This reminded us of the funny Mark Twain story about how "useful" mathematics was -- why, with mathematics, you could show that the Mississippi River was losing so much length each year by meanders straightening out in flood time, that by the year

1927 the Mississippi River would be only 2.5 miles long from headwaters to mouth! (We thought of this as we gazed at the river on our trip, too)

Jesse also came upon the notion of the absurdity of some ideas of "average" in the PHANTOM TOLLBOOTH when meeting the boy in the story who was only half there, cut right down the middle. Or, to put it more exactly, cut into .58 of a child, for he was the extra bit of a child from the average size family with "2.58 children." If your child finds all math ideas terribly serious and dull and tedious, try this book out on him!

Another book that gave the kids a real laugh AND some new mathematics thinking, was Mitsumasa Anno's book *Socrates and the Three Little Pigs*, a book about permutations and probability. This one is not a wordless picture book like many of Anno's, but has a delightful text following the thinking of the wolf, Socrates, as he tries to decide which house would be the most likely to have a little pig in it. Quite sophisticated math, done very graphically -- and with fine humor. We also are enjoying Anno's book *Sundials*, which goes into all the mathematics of time-telling, with the earth envisioned as a big sundial. Longitude, latitude, the movements and angles of shadows all become real things to experiment with. The book even has pop-up sundials all through it. (We keep this one away from Baby Hannah!)

Once you begin opening up to the world of math all around you, I guarantee that good books and resources and "natural" problems will spring up at you. Carl Sandburg even wrote poems about math, and most folks know that Lewis Caroll was not primarily a children's author, but was actually a mathematician. You'll be able to extend the textbook learnings into reality. Why, I've been amazed to find that even multiplication of fractions ACTUALLY happens -- it's not just something to plague fifth grade math students.

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[1]N. Juster. *The Phantom Tollbooth*, New York: Epstein & Carroll, distributed by Random House, page 175.
